Int. J. Solids Structures Vol. 19, No. 3, pp. 263–279, 1983
Printed in Great Britain.

0020–7683/83/030263–17$03.00/0
Pergamon Press Ltd.

IMPULSIVE LOADING OF A CYLINDRICAL SHELL WITH TRANSVERSE SHEAR AND ROTATORY INERTIA

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(*Received* 11 *January* 1982; *in revised form* 1 *June* 1982)

Abstract—Theoretical solutions are presented for the dynamic response of a simply supported cylindrical shell which is loaded impulsively and made from a rigid perfectly plastic material. The influence of rotatory inertia in the equilibrium equations is examined and plastic behaviour is controlled by a yield condition which retains the transverse shear force as well as the circumferential membrane force and longitudinal bending moment.

It appears that transverse shear effects exercise a less important influence on the maximum permanent transverse displacements of cylindrical shells with practical dimensions than found earlier for impulsively loaded beams and circular plates. Nevertheless, transverse shear effects do exercise an important influence on the partition of the initial kinetic energy which is absorbed in the various deformation modes.

NOTATION

c^2 defined by eqn (10c)
p lateral pressure (Fig. 1)
t time
w transverse displacement (Fig. 1)
w_1, w_f transverse displacement at supports and when motion ceases
$\bar{w}$ $w/\{\mu V_0^2 L^2/M_0\}$
$\dot{\bar{w}}$ $\dot{w}/V_0$
x axial coordinate (Fig. 1)
z axial location of interface
z_0 time-independent axial location of interface
H thickness of cylindrical shell
I_r $\mu H^2/12$
I^2 $I_r/\mu L^2$, non-dimensional rotatory inertia
$2L$ length of cylindrical shell
M bending moment per unit length (Fig. 1)
M_0 magnitude of bending moment per unit length required for plastic flow of cross-section when $Q = N_\theta = 0$
$\bar{M}$ M/M_0
N_θ circumferential membrane force per unit length (Fig. 1)
N_0 magnitude of membrane force per unit length required for plastic flow of cross-section when $M = Q = 0$
$\bar{N}$ N_θ/N_0
Q transverse shear force per unit length (Fig. 1)
Q_0 magnitude of transverse shear force per unit length required for plastic flow of cross-section when $M = N_\theta = 0$
$\bar{Q}$ Q/Q_0
R_M energy dissipated due to bending deformations non-dimensionalised with respect to $2\pi RL\mu V_0^2$
R_N energy dissipated due to membrane deformations non-dimensionalised with respect to $2\pi RL\mu V_0^2$
R_Q energy dissipated due to shearing deformations non-dimensionalised with respect to $2\pi RL\mu V_0^2$
$\bar{T}$ $M_0 t/\mu V_0 L^2$
T_f dimensionless response duration
V_0 initial uniformly distributed impulsive velocity
W, W_1 transverse displacement at interface or center and supports, respectively
$\dot{\bar{W}}$, $\dot{\bar{W}}_1$ $\dot{W}/V_0$, $\dot{W}_1/V_0$
α x/L
$\bar{\alpha}$ x/z
γ transverse shear strain
ϵ_θ circumferential membrane strain
κ_x, κ_θ longitudinal and circumferential curvature changes, respectively
λ z/L
λ_0 z_0/L

μ ρH
ν $Q_0 L/M_0$
ρ density
ψ defined in eqn (2a)
$(\dot{})$ $\partial(\ )/\partial t$ or $\partial(\ )/\partial T$

1. INTRODUCTION

Several studies have recently demonstrated the important influence of transverse shear effects on the dynamic plastic behaviour of structures. It was shown in Ref.[1] that a rigid plastic formulation which includes the influence of the transverse shear force in the yield criterion could be used to estimate the threshold impulse for a mode III transverse shear failure at the supports of a beam loaded impulsively[2]. Transverse shear effects play an important role in structures which respond with higher modal forms[3] and dominate the behaviour of ideal fibre-reinforced beams[4] and plates[5]. Indeed transverse shear effects have a more significant influence on the response of rigid plastic structures loaded dynamically than they do for the corresponding static cases[6].

Some remarks on the influence of transverse shear forces on the plastic yielding of beams is given in Ref.[7], while theoretical results for various dynamic beam problems which include both transverse shear and rotatory inertia effects are presented in Refs.[8–10]. A similar study for a simply supported rigid plastic circular plate loaded impulsively was reported in Ref[11]. It is the objective of this article to extend further the range of previous studies and examine the influence of transverse shear and rotatory inertia on the behaviour of cylindrical shells.

Haydl and Sherbourne[12, 13] have examined the influence of transverse shear on the plastic collapse of some cylindrical shells loaded statically. However, neither transverse shear nor rotatory inertia effects have been retained in any dynamic loading cases (e.g.[14–16], etc.). The particular case of a simply supported cylindrical shell subjected to a uniform impulse and undergoing infinitesimal displacements was examined in part of Ref.[17]. It may be shown that eqns (3) and (18) in Ref.[17] predict an infinitely large transverse shear force at the supports immediately motion commences. A theoretical analysis is developed herein for the same problem but with a finite transverse shear strength. A further analysis which includes the influence of rotatory inertia is also presented.

2. BASIC EQUATIONS

The equilibrium equations for the dynamic behaviour of the element of an axisymmetrically loaded cylindrical shell without axial load shown in Fig. 1 may be written in the form

$$\partial M/\partial x + Q - I_r \partial^2 \psi/\partial t^2 = 0 \qquad (1a)$$

and

$$\partial Q/\partial x + N_\theta/R - \mu \partial^2 w/\partial t^2 = -p \qquad (1b)$$

where $I_r = \mu H^2/12$, $\mu = \rho H$, $\partial w/\partial x = \psi + \gamma$, ψ is rotation of lines which were originally perpendicular to the initial mid-plane ($z = 0$) due to bending and

$$\gamma = \partial w/\partial x - \psi,\ \epsilon_\theta = -w/R,\ \kappa_x = \partial\psi/\partial x,\ \kappa_\theta \simeq 0 \qquad (2a\text{–}d)$$

are the transverse shear strain, circumferential strain, longitudinal and circumferential curvature changes, respectively. Equations (1) and (2) are consistent according to the principle of virtual work.

The dynamic continuity condition across a discontinuity front, which travels from region 1 to region 2 with a velocity c in a continuum having a density ρ, may be written[18]

$$[\sigma_{i1}] = -\rho c[\partial u_i/\partial t] \qquad (3)$$

where $[X] = X_2 - X_1$, and when the particle velocity $(\partial u_1/\partial t)$ in region 1, which is normal to the

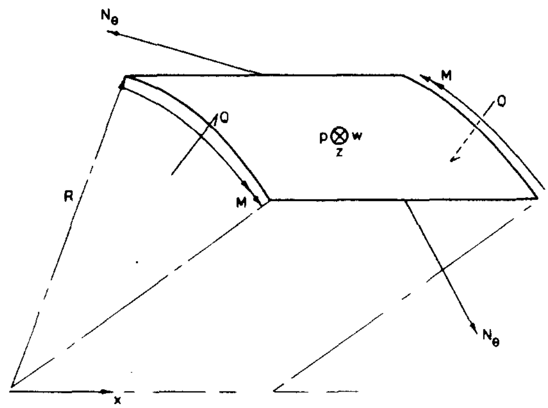


Fig. 1. Definition of generalised forces and moments and displacements for a cylindrical shell.

discontinuity front, is negligible compared with c. The displacements u_i act along the x_i axes with x_1 directed from region 1 to region 2 and normal to the discontinuity front.

Now, $x_1 = x$, $x_2 = R\theta$, $x_3 = z$, $\sigma_{11} = \sigma_x = 0$, $\sigma_{21} = \sigma_{\theta x} = 0$, $\sigma_{31} = \sigma_{zx}$, $u_1 = -z\psi$, $u_2 = 0$, $u_3 = w$ for the particular case of an axisymmetrically loaded cylindrical shell with the variables defined in Fig. 1 and in the Notation. Thus, if eqn (3) with i = 1 is multiplied by z and integrated with respect to x, then

$$[M] = -cI_r[\dot{\psi}], \tag{4a}$$

while eqn (3) with $i = 3$ when integrated with respect to z gives

$$[Q] = -\mu c[\dot{w}]. \tag{4b}$$

The kinematic continuity condition associated with eqn (3) is [18]

$$[\partial u_i/\partial t] = -c[\partial u_i/\partial x_1] \tag{5}$$

which using the variables for an axisymmetrically loaded cylindrical shell predicts

$$[\dot{\psi}] = -c[\partial\psi/\partial x] \tag{6a}$$

and

$$[\dot{w}] = -c[\partial w/\partial x]. \tag{6b}$$

3. IMPULSIVE LOADING OF A CYLINDRICAL SHELL WITH TRANSVERSE SHEAR

It was remarked in the Introduction that the transverse shear force at the simple supports of the impulsively loaded cylindrical shell examined in Ref. [17] is infinitely large at the start of motion. A theoretical analysis of the same problem is presented in this section but for a shell made from a rigid perfectly plastic material having a finite transverse shear strength.

Plastic flow in the theoretical analyses in Refs [14–17] is controlled by a square yield curve relating the circumferential membrane force N_θ and longitudinal bending moment M. A cube shaped yield surface is used in this article in order to assess the importance of transverse shear on the response of a shell. This yield surface is constructed by assuming that the square yield curve controls the interaction between N_θ and M for all values of the transverse shear force Q which is bounded by the inequality $-Q_0 \le Q \le Q_0$, where Q_0 is the transverse shear force per unit length required for independent plastic yielding of the shell cross-section.†

†Most theoretical studies on the static and dynamic plastic behaviour of structures have used simplified yield surfaces to assess the potential influence of transverse shear effects. More accurate yield surfaces are available (e.g., Illyushin-Shapiro), but no supporting experimental work.

3.1 *Class I shells*, $O \leq \nu \leq 2$

The dimensionless transverse velocity profile for this class of shells subjected to a uniformly distributed internal impulsive velocity V_0 is

$$\dot{w} = -\dot{W} \text{ for } 0 \leq \alpha \leq 1, \tag{7}$$

where the variables are defined in Fig. 2 and in the Notation. This velocity profile gives a transverse shear hinge at both supports as indicated in Fig. 2(b), while in the remainder of the shell, eqns (2) predict $\dot{\gamma} = 0$, $\dot{\epsilon} > 0$, $\dot{\kappa}_x = 0$. Thus, the normality requirements of plasticity theory demand $N_\theta = N_0$ which together with eqn (7) allows eqns (1) (with $I_r = p = 0$) to be integrated and give

$$\bar{M}(\alpha) = \nu\alpha(2-\alpha)/2, \quad \bar{Q}(\alpha) = \alpha - 1 \tag{8a, b}$$

and

$$\ddot{W} = -(\nu + 2c^2) \tag{9}$$

when satisfying the boundary conditions $M = 0$ and $Q = -Q_0$ at $x = 0$ and the symmetry requirement $Q = 0$ at $x = L$ and where,

$$\alpha = x/L, \quad \nu = Q_0 L/M_0, \quad c^2 = N_0 L^2/2M_0 R, \quad \bar{M} = M/M_0,$$

$$\bar{Q} = Q/Q_0, \quad \dot{w} = \dot{w}/V_0, \text{ and } (\dot{\ }) = \partial(\ )/\partial T \text{ with}$$

$$T = M_0 t/\mu V_0 L^2. \tag{10a–h}$$

Equation (9) may be integrated and with the initial condition $\dot{w} = -V_0$, gives

$$\dot{W} = 1 - (\nu + 2c^2)T \tag{11}$$

which shows that motion terminates when

$$T_1 = (\nu + 2c^2)^{-1}. \tag{12}$$

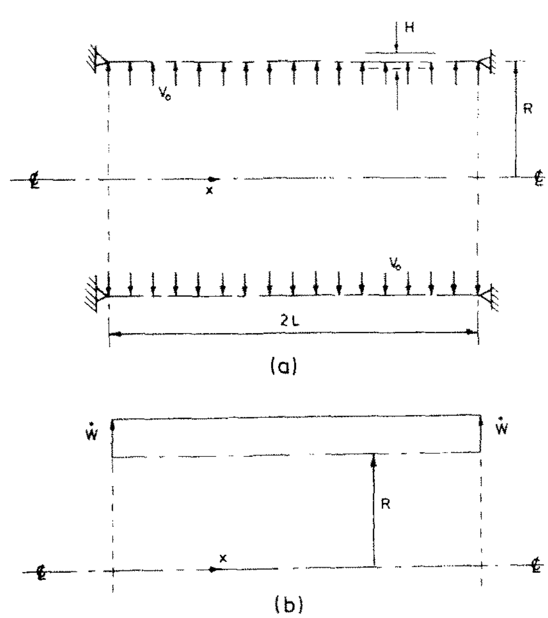


Fig. 2. (a) Impulsively loaded cylindrical shell. (b) Axisymmetric velocity profile for Class I cylindrical shells.

A further integration of eqns (7) and (11) yields the permanent dimensionless transverse displacement

$$\bar{w}(T_1) = -(\nu + 2c^2)^{-1}/2. \qquad (13)$$

$\bar{Q}(\alpha)$ given by eqn (8b) is statically admissible for $0 \le \alpha \le 1$, while $\bar{M}(\alpha)$ according to eqn (8a) is statically admissible provided $\nu \le 2$ in order to avoid a yield violation at $\alpha = 1$. Thus, the foregoing analysis is kinematically and statically admissible and is therefore exact according to the theory of plasticity for the selected yield surface.

It may be shown that the proportions of the initial kinetic energy dissipated in shearing and membrane deformations are $\nu(\nu + 2c^2)^{-1}$ and $2c^2(\nu + 2c^2)^{-1}$, respectively.

The dimensionless transverse displacement $\bar{w}(T_1)$ is manifested as a shear slide at the supports which must not become too large to avoid failure of a shell. A failure criterion which is suitable for engineering purposes was developed in Ref. [1] for beams and may be written for the present case in the form

$$w(t_1) \le kH \qquad (14)$$

where $0 \le k \le 1$ and H is the shell thickness.

3.2 *Class II shells*, $2 \le \nu \le 3$

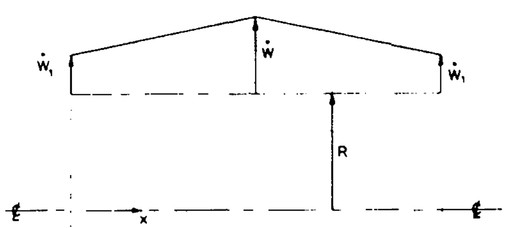
If $\nu \ge 2$, then eqn (8a) shows M violates the yield condition at the shell centre ($x = L$). This suggests that a plastic hinge forms at $x = L$ as shown in the axisymmetric velocity profile which is sketched in Fig. 3(a). This phase of motion is completed when shear sliding ceases at the supports and is followed by the second and final stage of motion sketched in Fig. 3(b).

3.2.1 *First phase of motion*, $0 \le T \le T_1$. The transverse velocity profile in Fig. 3(a) is

$$\dot{w} = -\dot{W}_1 - (\dot{W} - \dot{W}_1)\alpha \quad \text{for} \quad 0 \le \alpha \le 1 \qquad (15)$$

which predicts $\dot{\kappa}_x = 0$ and $\dot{\epsilon}_\theta \ge 0$ according to eqns (2) with $\dot{W}_1 \le \dot{W}$ and $\gamma = 0$ for $0 < \alpha \le 1$. The normality rule of incremental plasticity therefore requires

$$N_\theta = N_0, \ -M_0 \le M \le M_0, \ \text{and} \ -Q_0 \le Q \le Q_0. \qquad (16a\text{–}c)$$



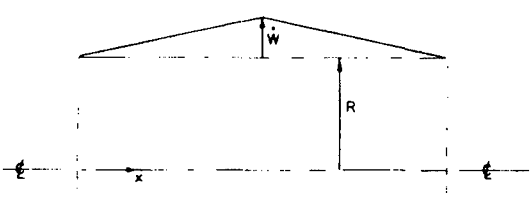


Fig. 3. (a) Axisymmetric velocity profile for first stage of motion of Class II cylindrical shells. (b) Axisymmetric velocity profile for second stage of motion of Class II cylindrical shells.

Equations (15), (16a) and (1a, b) with $I_r = p = 0$ together with the boundary conditions $\bar{M}(0) = \bar{Q}(1) = 0$, $\bar{M}(1) = 1$, $\bar{Q}(0) = -1$ give

$$\ddot{\bar{W}}_1 = 6 - 4\nu - 2c^2, \quad \ddot{\bar{W}} = -6 + 2\nu - 2c^2 \tag{17a, b}$$

$$\bar{Q}(\alpha) = -1 - \{3(\nu - 2)\alpha^2 + 2(3 - 2\nu)\alpha\}/\nu \tag{18}$$

and

$$\bar{M}(\alpha) = (\nu - 2)\alpha^3 + (3 - 2\nu)\alpha^2 + \nu\alpha. \tag{19}$$

Equations (17) may be integrated with respect to time to give the velocity field with $\dot{\bar{W}}_1(0) = \dot{\bar{W}}(0) = 1$ from which it is found that shear sliding ceases at the supports ($\dot{\bar{W}}_1(T_1) = 0$) when

$$T_1 = (4\nu + 2c^2 - 6)^{-1} \tag{20}$$

but

$$\dot{\bar{W}}(T_1) = 3(\nu - 2)(2\nu + c^2 - 3)^{-1}. \tag{21}$$

A further integration and the initial conditions $\bar{W}(0) = \bar{W}_1(0) = 0$ yields

$$\bar{W}_1(T_1) = (2\nu + c^2 - 3)^{-1}/4 \tag{22a}$$

and

$$\bar{W}(T_1) = -(9 - c^2 - 5\nu)(2\nu + c^2 - 3)^{-2}/4 \tag{22b}$$

where

$$\bar{w}(T_1) = -\bar{W}_1 - (\bar{W} - \bar{W}_1)\alpha. \tag{22c}$$

The total energy dissipated due to shearing deformations is

$$R_Q = \nu(4\nu + 2c^2 - 6)^{-1} \tag{23}$$

when non-dimensionalised with respect to the initial kinetic energy $2\pi RL\mu V_0^2$.

It may be shown that the generalised stress fields (18) and (19) are statically admissible provided $2 \leq \nu \leq 3$.

3.2.2 *Second phase of motion*, $T_1 \leq T \leq T_f$. It was observed in the previous section that the transverse velocity ceases at the supports at T_1 while the remainder of the shell continues in motion (eqn 21). This suggests the velocity field illustrated in Fig. 3(b) which is given by eqn (15) with $\dot{\bar{W}}_1 = 0$. Thus, eqns (16) still hold so that eqns (1) predict

$$\ddot{\bar{W}} = -3(c^2 + 1) \tag{24a}$$

$$\bar{Q}(\alpha) = (1 - \alpha)\{c^2 - 3 - 3(c^2 + 1)\alpha\}/2\nu \tag{24b}$$

and

$$\bar{M}(\alpha) = -c^2\alpha(\alpha - 1)^2/2 - \alpha(\alpha^2 - 3)/2 \tag{24c}$$

since

$$\bar{M}(0) = \bar{Q}(1) = 0 \text{ and } \bar{M}(1) = 1.$$

If eqn (24a) is integrated and made continuous with eqn (21) at T_1 then it is found that motion ceases when

$$T_f = (1+c^2)^{-1}/2 \tag{25a}$$

and the accompanying maximum permanent transverse displacement at $\alpha = 1$ is

$$\bar{W}_f(T_f) = (c^2+3\nu-5)(c^2+1)^{-1}(2\nu+c^2-3)^{-1}/4. \tag{25b}$$

It may be shown that eqns (24b) and (24c) are statically admissible when $2 \le \nu \le 3$, provided

$$c^2 \le 3 + 2\sqrt{3} \tag{26}$$

and that energy is conserved in both stages of motion.

3.3 *Class III shells,* $\nu \ge 3$

The velocity field in Fig. 4(a–c) with three phases of motion is required when $\nu \ge 3$ in order to avoid the yield violations that would occur with the velocity field in Fig. 3.

3.3.1 *First phase of motion.* $0 \le T \le T_1$. A stationary axisymmetric circular hinge forms at an axial distance z from each support (i.e. at $x = z$ and $x = 2L - z$). However, only one half of the shell is considered henceforth due to symmetry about the plane at $x = L$.

The velocity profile in Fig. 4(a) may be written in the dimensionless form

$$\dot{w} = -(\dot{W} - \dot{W}_1)\bar{\alpha} - \dot{W}_1, \quad 0 \le \bar{\alpha} \le 1 \tag{27a}$$

and

$$\dot{w} = -\dot{W}, \quad 1 \le \bar{\alpha} \le 1/\lambda \tag{27b}$$

where

$$\bar{\alpha} = x/z \quad \text{and} \quad \lambda = z/L. \tag{27c, d}$$

The flow rule of plasticity again requires eqns (16).

Now, eqns (1) with $I_r = p = 0$ and eqns (16) and (27b) for the central region give

$$\ddot{W} = -2c^2 \tag{28}$$

if $Q = 0$ and $M = M_0$ at $x = z$ and $Q = 0$ at $x = L$. It turns out that $\bar{Q} = 0$ and $\bar{M} = 1$ throughout the central zone. Equation (27a) governs the outer regions and when substituted into eqns (1) with eqns (16) and (28) eventually leads to

$$\ddot{W}_1 = -2(3c^2+\nu^2)/3 \quad \text{and} \quad \lambda = 3/\nu \tag{29a, b}$$

when using† $\bar{M}(0) = \bar{Q}(1) = 0$, $\bar{M}(1) = 1$ and $\bar{Q}(0) = -1$. The associated generalised stress fields in the region $0 \le \bar{\alpha} \le 1$ are

$$\bar{Q}(\bar{\alpha}) = -(1-\bar{\alpha})^2 \tag{30a}$$

and

$$\bar{M}(\bar{\alpha}) = \bar{\alpha}(\bar{\alpha}^2 - 3\bar{\alpha} + 3) \tag{30b}$$

which are both statically admissible.

†$\bar{\alpha} = 1$ is the stationary hinge and $\bar{M}$ and $\bar{Q}$ have been made continuous between the central and outer regions.

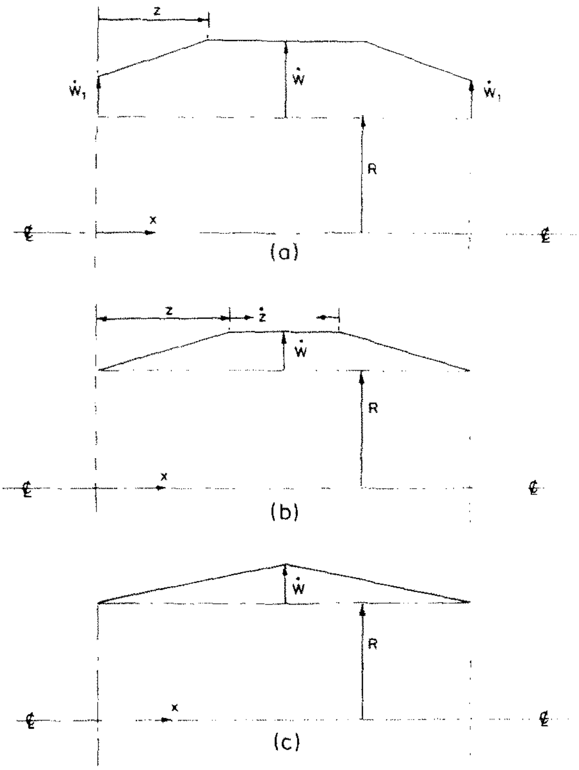


Fig. 4. Axisymmetric velocity profiles for (a) first, (b) second and (c) third stages of motion for Class III cylindrical shells.

Integrating eqn (29a) with respect to time gives

$$\dot{\bar{W}}_1 = 1 - 2(3c^2 + \nu^2)T/3 \tag{31}$$

when using the initial condition $\dot{\bar{W}}_1(0) = 1$. Thus, shear sliding ceases at the supports when

$$T_1 = 3(3c^2 + \nu^2)^{-1}/2 \tag{32}$$

and has an associated dimensionless transverse displacement

$$\bar{w}_1(T_1) = -3(3c^2 + \nu^2)^{-1}/4. \tag{33}$$

It may also be shown that the dimensionless transverse velocity and displacement at the shell centre are

$$\dot{\bar{w}}_1(T_1) = -\nu^2/(3c^2 + \nu^2) \tag{34a}$$

and

$$\bar{w}(T_1) = -3(3c^2 + 2\nu^2)(3c^2 + \nu^2)^{-2}/4, \tag{34b}$$

respectively.

3.3.2 *Second phase of motion*, $T_1 \le T \le T_2$. No transverse shear displacements occur during this phase of motion illustrated in Fig. 4(b) because shear sliding ceased at the supports when $T = T_1$. However, the axisymmetric circular plastic hinges at $\bar{\alpha} = 1$ are no longer stationary and travel towards the shell centre. In fact, this phase of motion is similar to the first phase in the theoretical analysis reported in Ref.[17] without shear deformations and for infinitesimal displacements except that the travelling hinges originate at $\bar{\alpha} = 1$ rather than at the supports.

Thus, when making due allowance for the initial conditions given by the behaviour at the end of the first phase of motion it may be shown that

$$\overset{\circ}{\lambda} = (3 + c^2\lambda^2)(1 - 2c^2T)^{-1}/\lambda \tag{35a}$$

and

$$T = \lambda^2(c^2\lambda^2 + 3)^{-1}/2, \tag{35b}$$

while for the region $0 \le \bar{\alpha} \le 1$,

$$\bar{Q}(\bar{\alpha}) = \{3(c^2\lambda^2 + 1)\bar{\alpha}^2 - 4c^2\lambda^2\bar{\alpha} + c^2\lambda^2 - 3\}/2\nu\lambda \tag{35c}$$

and

$$\bar{M}(\bar{\alpha}) = -(c^2\lambda^2 + 1)\bar{\alpha}^3/2 + c^2\lambda^2\bar{\alpha}(2\bar{\alpha} - 1)/2 + 3\bar{\alpha}/2 \tag{35d}$$

and $\bar{M} = 1$ and $\bar{Q} = 0$ in the central region $1 \le \bar{\alpha} \le \lambda^{-1}$.

It may be shown that eqns (35c, d) are statically admissible provided

$$c^2 \le 6 + 3\sqrt{5}. \tag{36}$$

This phase of motion is completed when the two travelling hinges coalesce at the centre which occurs at

$$T_2 = (c^2 + 3)^{-1}/2 \tag{37}$$

which is obtained from eqn (35b) with $\lambda = 1$. The corresponding dimensionless central transverse displacement is

$$\bar{w}(T_2) = -(6 + c^2)(3 + c^2)^{-2}/4 \tag{38}$$

while the associated velocity is

$$\dot{\bar{w}}(T_2) = -3(c^2 + 3)^{-1}. \tag{39}$$

3.3.3 *Third phase of motion*, $T_2 \le T \le T_f$. The transverse velocity profile in Fig. 4(c) with stationary plastic hinges governs the response during the final phase of motion and is similar to that shown in Fig. 3(b) for the second phase of motion of a class II cylindrical shell considered in Section 3.2.2. It is also similar to the second phase of motion of the theoretical analysis in Ref. [17] for a cylindrical shell undergoing infinitestimal deflections without transverse shear effects.

It is straightforward to show when making the displacement and velocity profiles continuous at T_2 that motion ceases at

$$T_f = (c^2 + 1)^{-1}/2, \tag{40}$$

while the associated permanent transverse displacement at the shell centre ($x = L$) accumulated during this phase of motion is

$$\bar{w}(T_f - T_2) = -3(c^2 + 1)^{-1}(c^2 + 3)^{-2}/2. \tag{41}$$

Thus, the maximum permanent dimensionless transverse displacement of the cylindrical shell is given by the sum of eqns (38) and (41).

The generalised stresses are statically admissible provided

$$c^2 \leq 9 \tag{42}$$

which is more restrictive than inequality (36).

The final permanent transverse displacement profile is

$$\bar{w}(T_f) = -3(3c^2+\nu^2)^{-1}/4 - 3(x/L)[(c^2+1)^{-1} - \nu(3c^2+\nu^2)^{-1}$$

$$+\tan^{-1}(c/\sqrt{3})/(\sqrt{(3)}c) - \tan^{-1}(\sqrt{(3)}c/\nu)/(\sqrt{(3)}c)]/8,\ 0 \leq x/L \leq 3/\nu \tag{43a}$$

and

$$\bar{w}(T_f) = -1/4c^2 + 9\{3+(cx/L)^2\}^{-2}/4c^2 - 3(x/L)[(c^2+1)^{-1}$$

$$+\tan^{-1}(c/\sqrt{3})/(\sqrt{(3)}c) - \tan^{-1}(cx/\sqrt{(3)}L)/(\sqrt{(3)}c)$$

$$-\{5+(cx/L)^2\}\{3+(cx/L)^2\}^{-2}(x/L)]/8,\ 3/\nu \leq x/L \leq 1. \tag{43b}$$

4. IMPULSIVE LOADING OF A CYLINDRICAL SHELL WITH TRANSVERSE SHEAR AND ROTATORY INERTIA

It is the object of this section to repeat the theoretical analysis in Section 3 for a cylindrical shell with rotatory inertia (I_r) effects.

4.1 *Class I shells*, $0 \leq \nu \leq 2$

It is evident from eqn (7) and Fig. 2(b) that the rotatory inertia term in eqn (1a) is zero when $0 \leq \nu \leq 2$ so that the theoretical analysis and results in Section 3.1 remain valid whether rotatory inertia is retained or not.

4.2 *Class II shells*, $\nu \geq 2$

It may be shown that the transverse velocity field illustrated in Fig. 3(a) gives rise to a yield violation at $\alpha = 1$ for the $I_r \neq 0$ case. Thus, it is assumed that a plastic zone develops in a central region with $N_\theta = N_0$, $M = M_0$ and no transverse shear effects (i.e. $\gamma = 0$). In this circumstance, $\psi = \partial w/\partial x$ and eqns (1) with $p = 0$ give

$$\partial^2 \ddot{w}/\partial x^2 - \mu\ddot{w}/I_r = -N_0/RI_r \tag{44a}$$

the solution of which is

$$\dot{w} = A \sinh\{(\mu/I_r)^{1/2}x\} + B\cosh\{(\mu/I_r)^{1/2}x\} + N_0 t/\mu R + C, \tag{44b}$$

where A and B are time-dependent constants of integration and C is an arbitrary function of x.

4.2.1 *First phase of motion*, $0 \leq T \leq T_1$. Now the transverse velocity field in Fig. 5(a), which is linear in the region $0 \leq x \leq z_0$ and governed by eqn (44b) in the stationary central plastic zone, can be written in the dimensionless form

$$\dot{\bar{w}} = -(\dot{\bar{W}} - \dot{\bar{W}}_1)\bar{\alpha}_0 - \dot{\bar{W}}_1,\ 0 \leq \bar{\alpha}_0 \leq 1 \tag{45a}$$

and

$$\dot{\bar{w}} = -(\dot{\bar{W}} - 1 + 2c^2T)\cosh\{(1-\bar{\alpha}_0\lambda_0)/I\}/\cosh\{(1-\lambda_0)/I\} + 2c^2T - 1 \tag{45b}$$

for $1 \leq \bar{\alpha}_0 \leq \lambda_0^{-1}$, where

$$\bar{\alpha}_0 = x/z_0,\ \lambda_0 = z_0/L \text{ and } I^2 = I_r/\mu L^2. \tag{45c–e}$$

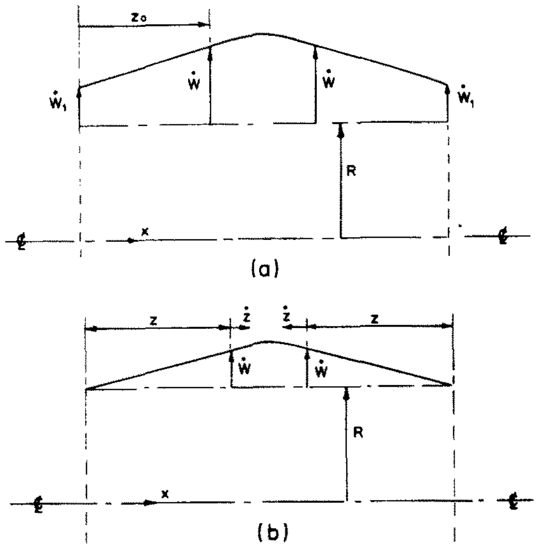


Fig. 5. (a) Axisymmetric velocity profile for first stage of motion of an impulsively loaded cylindrical shell with $I_r \neq 0$ and $\nu \geq 2$. (b) Velocity profile for second stage of motion.

Equations (1) and (45) predict the generalised stresses,

$$\bar{N}(\bar{\alpha}_0, T) = 1, \tag{46a}$$

$$\bar{M}(\bar{\alpha}_0, T) = c^2\bar{\alpha}_0^{\,2}\lambda_0^{\,2} + (\ddot{W} - \ddot{W}_1)\bar{\alpha}_0^{\,3}\lambda_0^{\,2}/6 + \ddot{W}_1\bar{\alpha}_0^{\,2}\lambda_0^{\,2}/2 + \nu\bar{\alpha}_0\lambda_0 - I^2(\ddot{W} - \ddot{W}_1)\bar{\alpha}_0 \tag{46b}$$

and

$$\bar{Q}(\bar{\alpha}_0, T) = -2c^2\bar{\alpha}_0\lambda_0/\nu - (\ddot{W} - \ddot{W}_1)\bar{\alpha}_0^{\,2}\lambda_0/2\nu - \ddot{W}_1\bar{\alpha}_0\lambda_0/\nu - 1 \tag{46c}$$

for $0 \leq \bar{\alpha}_0 \leq 1$, and

$$\bar{N}(\bar{\alpha}_0, T) = \bar{M}(\bar{\alpha}_0, T) = 1 \tag{46d, e}$$

and

$$\bar{Q}(\bar{\alpha}_0, T) = I(\ddot{W} + 2c^2)[\sinh\{(1 - \bar{\alpha}_0\lambda_0)/I\}/\cosh\{(1 - \lambda_0)/I\}]/\nu \tag{46f}$$

when $1 \leq \bar{\alpha}_0 \leq \lambda_0^{-1}$ with $\dot{w}(\bar{\alpha}_0, 0) = -1$, $\ddot{w}(0, T) = -\ddot{W}_1$, $\ddot{w}(1, T) = -\ddot{W}$, $\bar{Q}(\lambda_0^{-1}, T) = 0$, $\bar{M}(0, T) = 0$ and $\bar{Q}(0, T) = -1$.

The continuity requirements $[\bar{M}(1, T)] = 0$ and $[\partial\bar{M}(1, T)/\partial\bar{\alpha}_0] = 0$† at the boundary of the central plastic zone give

$$\ddot{W}_1 = -\{2\lambda_0^{\,2}(c^2\lambda_0^{\,2} + 2\nu\lambda_0 - 3) + 12I^2(1 + c^2\lambda_0^{\,2})\}/\{\lambda_0^{\,2}(\lambda_0^{\,2} + 6I^2)\} \tag{47a}$$

and

$$\ddot{W} = \{2\lambda_0^{\,2}(\nu\lambda_0 - 3 - c^2\lambda_0^{\,2}) - 12I^2(1 + c^2\lambda_0^{\,2})\}/\{\lambda_0^{\,2}(\lambda_0^{\,2} + 6I^2)\}. \tag{47b}$$

In addition $[\bar{Q}(1, T)] = 0$ yields

$$\tan\{(1 - \lambda_0)/I\} = \{3I\lambda_0(\nu\lambda_0 - 2)\}\{3\lambda_0^{\,2} - \nu\lambda_0^{\,3} + 6I^2\}^{-1} \tag{48}$$

from which the location of the interface (λ_0) can be obtained.

†It may be shown when using eqn (1a) that $[\bar{Q}(1, T)] = 0$ may be replaced by the requirement $[\partial\bar{M}(1, T)/\partial\bar{\alpha}_0] = 0$ provided $[\partial^3\bar{w}(1, T)/\partial\bar{\alpha}_0\partial T^2] = 0$.

It may be shown when solving eqns (47) with the initial conditions $\dot{\bar{W}}_1(0) = \dot{\bar{W}}(0) = 1$ and $\bar{W}_1(0) = \bar{W}(0) = 0$ that shear sliding ceases at the simple supports when

$$T_1 = \lambda_0^2(\lambda_0^2 + 6I^2)\{2\lambda_0^2(c^2\lambda_0^2 + 2\nu\lambda_0 - 3) + 12I^2(1 + c^2\lambda_0^2)\}^{-1}, \tag{49}$$

while the accompanying dimensionless transverse displacements are

$$\bar{w}(0, T_1) = -\lambda_0^2(\lambda_0^2 + 6I^2)\{4\lambda_0^2(c^2\lambda_0^2 + 2\nu\lambda_0 - 3) + 24I^2(1 + c^2\lambda_0^2)\}^{-1} \tag{50a}$$

$$\bar{w}(1, T_1) = -\lambda_0^2(\lambda_0^2 + 6I^2)\{c^2\lambda_0^4 + 5\nu\lambda_0^3 - 9\lambda_0^2 + 6I^2(1 + c^2\lambda_0^2)\}$$
$$\{2\lambda_0^2(c^2\lambda_0^2 + 2\nu\lambda_0 - 3) + 12I^2(1 + c^2\lambda_0^2)\}^{-2} \tag{50b}$$

and

$$\bar{w}(\lambda_0^{-1}, T_1) = -[(\nu\lambda_0^3 - 3\lambda_0^2 - 6I^2)\,\text{sech}\,\{(1 - \lambda_0)/I\}]T_1^2/\{\lambda_0^2(\lambda_0^2 + 6I^2)\} + c^2T_1^2 - T_1 \tag{50c}$$

at the supports, interface and centre, respectively. $(\ddot{\bar{W}}_1 - \ddot{\bar{W}}) < 0$, $\ddot{\bar{W}} < 0$ and the generalized stresses are statically admissible provided

$$2\lambda_0^{-1} \le \nu \le 3\lambda_0^{-1} + 6I^2\lambda_0^{-3}. \tag{51†}$$

4.2.2. *Final phase of motion*, $T_1 \le T \le T_f$. The behaviour during this phase of motion is governed by the transverse velocity profile illustrated in Fig. 5(b) with no shear sliding at the supports and a central plastic zone with a time-dependent boundary. Thus,

$$\dot{\bar{w}} = -\dot{\bar{W}}\bar{\alpha}, \quad 0 \le \bar{\alpha} \le 1 \tag{52a}$$

and

$$\dot{\bar{w}} = -(\dot{\bar{W}} - 1 + 2c^2T)[\cosh\{(1 - \bar{\alpha}\lambda)/I\}/\cosh\{(1 - \lambda)/I\}] + 2c^2T - 1$$
$$1 \le \bar{\alpha} \le \lambda^{-1} \tag{52b}$$

when using eqn (44b) and where $\bar{\alpha}$ and λ are defined by eqns (27c) and (27d), respectively. The requirement that $[\partial\dot{\bar{w}}(1, T)/\partial\bar{\alpha}] = 0$‡ yields

$$\dot{\bar{W}} = \lambda(1 - 2c^2T)\tanh\{(1 - \lambda)/I\}/[I + \lambda\tanh\{(1 - \lambda)/I\}]. \tag{53}$$

Thus, for $1 \le \bar{\alpha} \le \lambda^{-1}$, eqns (1) and (52b) may be solved to give the generalised stresses

$$\bar{N}(\bar{\alpha}, T) = \bar{M}(\bar{\alpha}, T) = 1 \tag{54a, b}$$

and

$$\bar{Q}(\bar{\alpha}, T) = (I/\nu)(\ddot{\bar{W}} + 2c^2)[\sinh\{(1 - \bar{\alpha}\lambda)/I\}/\cosh\{(1 - \lambda)/I\}]$$
$$+ (\dot{\lambda}/\nu)(\dot{\bar{W}} - 1 + 2c^2T)[\sinh\{(1 - \bar{\alpha}\lambda)/I\}/\{\cosh(1 - \lambda)/I\}]\tanh\{(1 - \lambda)/I\}, \tag{54c}$$

since $\bar{Q}(\lambda^{-1}, T) = 0$, Equations (1) and (52a) for $0 \le \bar{\alpha} \le 1$ together with $\bar{M}(0, T) = 0$ and $[\bar{Q}(1, T)] = 0$ give

$$\bar{N}(\bar{\alpha}, T) = 1 \tag{55a}$$

†This requirement is necessary to obtain a real root for λ_0 from eqn (48) and was satisfied in all the calculations presented herein. The upper bound on ν may be relaxed, but it is then necessary to ensure $\ddot{\bar{W}} < 0$ and that eqns (46b,c,f) are statically admissible.

‡Equation (4a) gives $[\dot{\psi}] = 0$ when $\dot{z} \ne 0$ and $I_r \ne 0$, or $[\partial\dot{w}/\partial x] = 0$ if $\dot{\gamma} = 0$.

$$\bar{M}(\bar{\alpha}, T) = c^2\lambda^2\bar{\alpha}(\bar{\alpha}-2) + \lambda^3\bar{\alpha}[\bar{\alpha}^2/6 - 1/2 - (I/\lambda)\tanh\{(1-\lambda)/I\} - I^2/\lambda^2]\,\mathrm{d}(\dot{\bar{W}}/\lambda)/\mathrm{d}T - 2I\bar{\alpha}\lambda c^2\tanh\{(1-\lambda)/I\} \tag{55b}$$

and

$$\bar{Q}(\bar{\alpha}, T) = -2c^2\lambda(\bar{\alpha}-1)/\nu - (\lambda/\nu)[\lambda(\bar{\alpha}^2-1)/2 - I\tanh\{(1-\lambda)/I\}]\,\mathrm{d}(\dot{\bar{W}}/\lambda)/\mathrm{d}T + 2c^2I\nu^{-1}\tanh\{(1-\lambda)/I\}, \tag{55c}$$

where

$$\mathrm{d}(\dot{\bar{W}}/\lambda)/\mathrm{d}T = -\lambda^{-1}[1 + c^2\lambda^2 + 2Ic^2\lambda\tanh\{(1-\lambda)/I\}][\lambda^2/3 + I^2 + \lambda I\tanh\{(1-\lambda)/I\}]^{-1} \tag{55d}$$

is obtained from $[\bar{M}(1, T)] = 0$.

The dimensionless transverse displacement accumulated during the second phase only is

$$\bar{w}(\bar{\alpha}\lambda, T) = -\int_{\lambda_0}^{\lambda} \dot{\bar{W}}\bar{\alpha}\,\mathrm{d}\lambda/\dot{\lambda} \tag{56a}$$

for the region $0 \le \bar{\alpha}\lambda \le \lambda_0$, and

$$\bar{w}(\bar{\alpha}\lambda, T) = \int_{\lambda_0}^{\bar{\alpha}\lambda} [I\mathring{\bar{W}}\cosh\{(1-\bar{\alpha}\lambda)/I\}/\{\lambda\sinh\{(1-\lambda)/I\}\} + 2c^2T - 1]\,\mathrm{d}\lambda/\mathring{\lambda} - \int_{\bar{\alpha}\lambda}^{1} \dot{\bar{W}}\bar{\alpha}\,\mathrm{d}\lambda/\dot{\lambda} \tag{56b}$$

for $\lambda_0 \le \bar{\alpha}\lambda \le 1$, where $\lambda = 1$ when motion ceases.

It may be shown that eqn (55b) gives $\partial\bar{M}/\partial\bar{\alpha} > 0$ at $\bar{\alpha} = 1$ as well as $\bar{M} = 0$ at $\bar{\alpha} = 0$ and $\bar{M} = 1$ at $\bar{\alpha} = 1$. However, eqn (55b) must be checked numerically to ensure $|\bar{M}(\bar{\alpha}, T)| \le 1$ within the region $0 < \bar{\alpha} < 1$. Equation (55c) is statically admissible provided $|\bar{Q}(0, T)| \le 1$, $|\bar{Q}(1, T)| \le 1$ and

$$c^2\lambda^4(c^2\lambda^2-6)/18 - \lambda^2/6 - 2I^2c^2\lambda^2(2 + c^2\lambda^2/3 - c^2) + I(c^4\lambda^5/3 - 2I^2c^4\lambda^3 - 2c^2\lambda^3 - 6I^2c^2\lambda - \lambda)\tanh\{(1-\lambda)/I\} + 2I^2c^2\lambda^2(c^2\lambda^2/3 - 1 - 2I^2c^2)\tanh^2\{(1-\lambda)/I\} - 2I^2 < 0. \tag{57}$$

The static admissibility of eqn (54c) must be checked numerically.

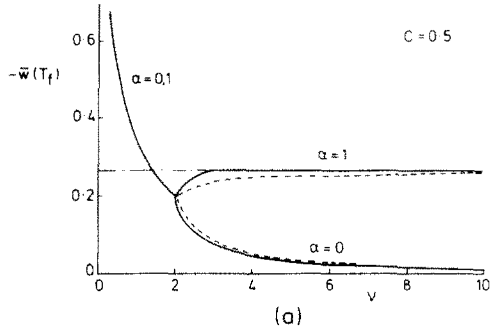
5. DISCUSSION

The theoretical analysis in Section 3 with $I = 0$ and a finite transverse shear strength ($\nu < \infty$) is compared in Figs. 6 and 9 with the theoretical predictions of eqn (31) in Ref.[17] which does not retain transverse shear ($\nu = \infty$), rotatory inertia ($I = 0$) and finite-deflection effects. In fact, eqn (43b) here is independent of ν and may be shown to be identical to eqn (31) in Ref.[17]. The maximum permanent transverse displacement from eqn (43b) with $\nu \ge 3$ and $x = L$ is

$$\bar{w}(T_f) = -(4 + c^2)(3 + c^2)^{-1}(1 + c^2)^{-1}/4 \tag{58}$$

which is identical to the result given by eqn (32) in Ref.[17] when due allowance is made for the difference in notation. It is stated in Ref.[17] that eqn (31) is valid when $c^2 \le 3$ but that the range of validity may be extended provided M is not smaller than $-M_0$ in the region $0 \le x \le L$. This exercise leads to the less restrictive requirement $c^2 \le 9$ according to eqn (42) here.

It is evident from Figs. 6 and 9 that transverse shear effects play an important role when ν is small as expected. However, the results in Figs. 6 and 9 with $I = 0$ and $\nu > 5$, approximately,



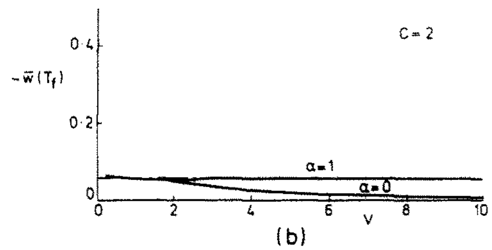


Fig. 6. Variation of dimensionless permanent transverse displacements at mid-length ($\alpha = 1$) and simple supports ($\alpha = 0$) an impulsively loaded cylindrical shell for (a) $c = 0.5$ and (b) $c = 2$.--- Bending only solution (eqn 58). ——$I = 0$. ---$I^2 = 1/3\nu^2$ for a solid homogeneous cross-section.

are similar to the simple predictions in Ref.[17], although Fig. 7 reveals that a significant portion of the initial kinetic energy is dissipated through shearing deformations at the supports for larger values of ν.

If the shell cross-section is solid and homogeneous, then $M_0 = \sigma_0 H^2/4$, $N_0 = \sigma_0 H$ and $Q_0 = \sigma_0 H/2$ giving $\nu = 2L/H$ and $c^2 = \nu L/R$. Thus, $\nu = 2$ corresponds to a very short shell or ring with a total length $2L = 2H$ and $c = 1$ when $L = R/2$. On the other hand, transverse shear effects would be more important for a cylindrical shell having a sandwich† cross-section with an inner core of thickness h and shear yield stress τ_0 and thin exterior sheets of thickness t and tensile yield stress σ_0. If the inner core alone supports transverse shear forces, then $Q_0 = \tau_0 h$, while $M_0 = \sigma_0 t(h + t)$ if the exterior sheets alone provide bending resistance. Thus, $\nu = Q_0 L/M_0$ gives

$$\nu = 2\left(\frac{L}{H}\right)\left(\frac{\tau_0}{\sigma_0/2}\right)\left\{\frac{h/H}{1-(h/H)^2}\right\} \tag{59}$$

when $H = h + 2t$. A sandwich shell with $2L/H = 10$, $\sigma_0/2\tau_0 = 8$ and $h/H = 0.735$ (e.g. a 0.5 in. thick core with 0.1 in. sheets gives $h/H = 0.714$) gives $\nu = 2$ for which transverse shear effects are important according to the results in Figs. 6 and 7. However, eqn (10c) with $N_0 = 2\sigma_0 t$ gives

$$c^2 = \frac{2L^2}{RH(1 + h/H)} \tag{60}$$

which, with the above values, predicts $2L/R = 0.087$ when $c = 0.5$. The combination of these parameters for this particular case gives $R/H \simeq 115$.

These results indicate that transverse shear effects are less important for practical impulsively loaded cylindrical shells than for the impulsively loaded beams and circular plates which are examined in Refs.[8, 11], respectively. This is partly due to the influence of the circumferential membrane forces, as shown in Fig. 7, particularly for large values of c, which should be contrasted with Fig. 9 in Ref.[11] for circular plates where $R_Q = 1$ when $0 \leq \nu \leq 1.5$.

†Assumed to have same mass per unit area (μ) as a shell with a solid and homogeneous cross-section. Possible crushing of core is disregarded.

It is interesting to observe that the impulsively loaded beams and circular plates with simple supports and no rotatory inertia ($I = 0$) examined in Refs.[19] and [11] have three types of response depending on the magnitude of the transverse shear parameter ν in common with the present study on cylindrical shells. In fact, the velocity profiles for all three structural members are similar. When $\nu = Q_0L/2M_0$ ($2L$ is beam length), then the class I deformation profile illustrated in Fig. 2(b) here is found for beams with $0 \leq \nu \leq 1$, class II in Fig. 3 for $1 \leq \nu \leq 1.5$ and class III in Fig. 4 for beams with $\nu \geq 1.5$. Similarly, when $\nu = Q_0R/2M_0$ ($2R$ is diameter) for a circular plate, these three classes of behaviour are observed for $0 \leq \nu \leq 1.5$, $1.5 \leq \nu \leq 2$ and $\nu \geq 2$, respectively. In the present case for a cylindrical shell, classes I, II and III are associated

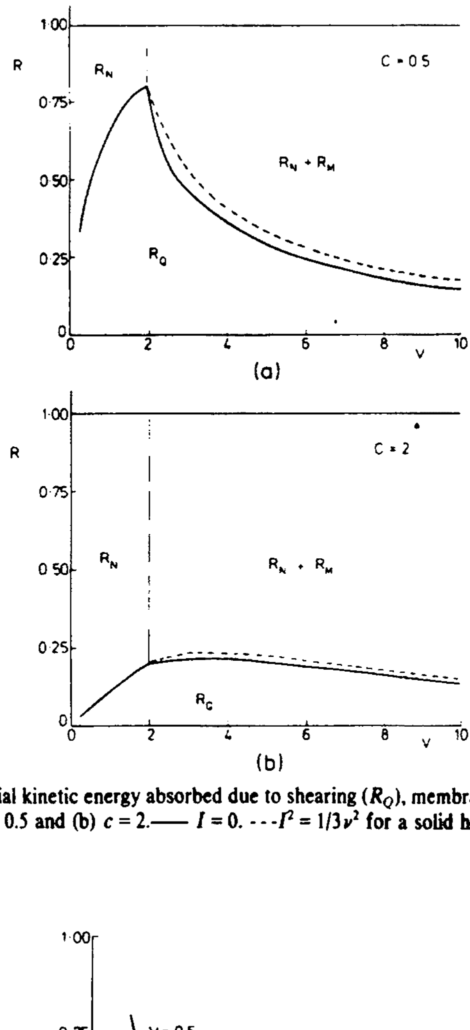


Fig. 7. Proportion of initial kinetic energy absorbed due to shearing (R_Q), membrane (R_N) and bending (R_M) deformations for (a) $c = 0.5$ and (b) $c = 2$.—— $I = 0$. - - - $I^2 = 1/3\nu^2$ for a solid homogeneous cross-section.

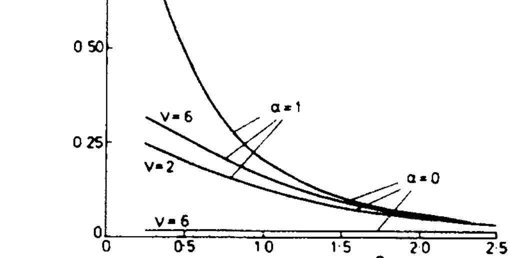


Fig. 8. Variation of dimensionless permanent transverse displacements at mid-length ($\alpha = 1$) and simple supports ($\alpha = 0$) of an impulsively loaded cylindrical shell with the shell parameter c for several values of ν and $I = 0$.

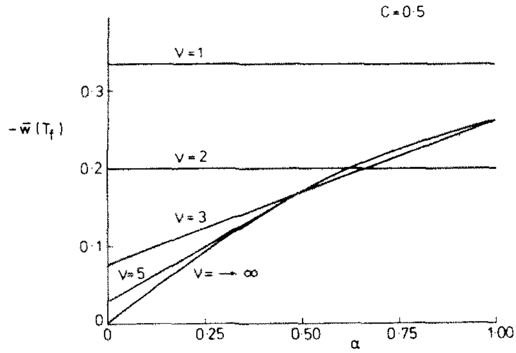


Fig. 9. Dimensionless permanent deformed profiles of impulsively loaded cylindrical shells with $c = 0.5$ and $I = 0$.

with the ranges $0 \leq \nu \leq 2$, $2 \leq \nu \leq 3$ and $\nu \geq 3$, respectively, where $\nu = Q_0L/M_0$ and $2L$ is shell length.

If rotatory inertia ($I \neq 0$) and transverse shear effects ($\nu \neq 0$) are included in the theoretical analyses then the velocity profiles in Fig. 5 for cylindrical shells with $\nu \geq 2$ are similar to the corresponding ones for beams with $\nu \geq 1$[8] and plates with $\nu \geq 1.5$[11].

However, although the velocity profiles for the three structural members considered above are similar, the theoretical predictions for the transverse displacements and other quantities are quite different because of the different equilibrium equations and generalised stresses contained in the yield criteria (M and Q in beam, M_r, M_θ and Q_r in circular plate and M, N_θ and Q in cylindrical shell).

It is evident from Fig. 6 that the inclusion of rotatory inertia (I) in the governing equations and the retention of transverse shear as well as bending and membrane effects in the yield criterion leads to an increase in the permanent transverse shear sliding at the shell supports and a decrease in the maximum final transverse displacement. However, the changes brought about by the consideration of I are relatively small so that the simpler theoretical analysis in Section 3 which includes transverse shear effects with $I = 0$ would probably suffice for most practical purposes. Even transverse shear effects are small for cylindrical shells with solid cross-sections and having practical dimensions, so that the theoretical predictions in Ref.[17] which retain only the influence of bending moments and membrane forces in the yield condition might be adequate for most designs, except possibly for shells with sandwich cross-sections. Notwithstanding these comments, transverse shear effects have an important influence on the proportion of energy dissipated in various modes as indicated in Fig. 7.

The duration of response of the analysis in Section 4 with $I \neq 0$ gives results identical to the corresponding theoretical predictions with $I = 0$ in Section 3. Indeed, eqn (40) for the response duration when $I = 0$ is identical to eqn (29) in Ref.[17] which does not retain transverse shear forces in the yield condition.

The amount of transverse shear sliding at the boundaries of cylindrical shells in all the theoretical analyses reported herein should not exceed a certain proportion of the shell thickness if severance is not to occur[1]. In addition, the material is assumed to be strain rate insensitive[6], and in order to remain consistent with an infinitesimal theory the difference between the maximum transverse displacements at the shell centre and transverse shear sliding at the supports should be less than the shell thickness, approximately[17]. Material elasticity is disregarded herein but requires further study because transverse shear effects are greatest in the early stages of motion when elastic effects are most important. Moreover, further work is required to extend the range of validity of the analysis to cater for longer shells with larger values of c.

6. CONCLUSIONS

Theoretical solutions for the dynamic response of a simply supported cylindrical shell which is impulsively loaded and made from a rigid perfectly plastic material are developed when either neglecting or retaining rotatory inertia in the equilibrium equations. Plastic flow is controlled by a yield condition which includes the transverse shear force as well as the circumferential membrane force and longitudinal bending moment.

It transpires that rotatory inertia effects are relatively unimportant so that the simpler analysis in Section 3 with $I = 0$ would probably suffice for most practical purposes. Furthermore, the calculations also indicate that transverse shear effects exercise a less important influence in the maximum permanent transverse displacements of practical cylindrical shells than previously found for impulsively loaded beams and circular plates. Nevertheless, transverse shear effects have a marked effect on the partition of the initial kinetic energy absorbed in the various modes.

Acknowledgements—The authors wish to record their gratitude to Dr. N. Perrone and Dr. N. Basdekas of the Structural Mechanics Programme of O.N.R. (Washington, D.C.) (contract number N00014-76-C-0195 Task NR 064 510) and the Department of Ocean Engineering at M.I.T. who supported the initial phase of this work. The later stage of this study was conducted in the Department of Mechanical Engineering at the University of Liverpool. The authors are indebted to Mrs. M. White for her typing and Mrs. A. Green for her preparation of the drawings, both from the Department of Mechanical Engineering in the University of Liverpool.

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